Ligand-Mediated Nanocluster Formation with Classical and Autocatalytic Growth

Mohsen Farshad, Dylan Suvlu, and Jayendran C. Rasaiah

Department of Chemistry, University of Maine, Orono, Maine 04469, United States

ABSTRACT: We present a systematic study of ligand-mediated nanocluster (NC) formation using a kinetic model, which provides atomic insight into sub-nanometer cluster (S-NC) and NC formation. Our model describes the role of ligand-mediated nucleation and growth in obtaining monodisperse NCs. Nucleation includes metal ion reduction, reversible ligand association to the metal ion/atom, and formation of dimer nuclei. Growth can occur through autocatalytic surface growth and ligand-associated monomer addition to the cluster, depending on the rate of metal ion to neutral metal atom conversion. Furthermore, we studied the effect of the initial concentration of metal ions on NC formation using fast and slow reducing agents in the presence of slowly and rapidly binding ligands. The model shows that fast nucleation, slow growth, and a high molar ratio of rapidly binding ligand to metal ions promote the formation of S-NCs and NCs. Our results can guide experiments in the synthesis of ultrasmall clusters.

Nanoclusters (NCs) are clusters of metal atoms with diameters smaller than 2 nm and contain unique properties because of their small sizes. Understanding their formation is useful for a variety of applications in catalysis, bio-imaging and sensing, and medical therapies. The synthesis of atomically precise NCs is possible with meticulous control over experimental conditions, which requires knowledge of the variables that affect their sizes. Advanced experimental techniques, such as liquid cell transmission electron microscopy, have allowed researchers to observe nanoparticle growth with unprecedented resolution. Theoretical models have also unveiled many factors that affect the size of nanoparticles. However, many key factors remain to be investigated. For example, experiments on silver and gold nanoparticles without a strongly binding ligand show rapid growth of nanoparticles, in one case approaching diameters greater than 20 nm in a matter of milliseconds. Experiments with strongly binding ligands (thiols or phosphine ligands), however, show stable NC synthesis with diameters less than 2 nm. Here, we take steps toward finding the optimal conditions for sub-nanometer cluster (S-NC) formation and provide mechanistic insight into their ligand-mediated nucleation and growth. To pursue our intention, we developed a ligand-mediated kinetic model to investigate the formation of metal NCs starting from individual atoms (monomers) to the final stages of formation.

For several years, nanoparticle nucleation and growth were explained by LaMer’s burst nucleation mechanism followed by nanoparticle growth. LaMer’s burst nucleation is based on Becker and Döring’s classical nucleation theory. In LaMer’s nucleation mechanism, the concentration of monomers reaches a critical supersaturation point after which they nucleate by overcoming the energy barrier for nucleation. This is followed by growth of nanoparticles as a separate step. However, experiments show that CNT fails to adequately describe nucleation and growth of transition-metal nanoparticles. Finke and Watzky (FW) considered nucleation and growth occurring simultaneously through a two-step mechanism. In the FW model, slow, continuous nucleation occurs simultaneously with autocatalytic growth, which is controlled by the reaction rates. Moza et al. developed a model of ligand-mediated nucleation and autocatalytic growth of nanoparticles, whereas Lazzari et al. reported a kinetic study of ligand-mediated CdSe nanoparticle formation. The authors fit their model to the experimental data and extracted temperature-dependent kinetic parameters.

We adapted the method of Lazzari et al. and developed a ligand-mediated model of NC formation in which we investigate the parameter space of NC formation. We explore the initial conditions and rate constants that allow the synthesis of stable S-NCs and NCs. Our kinetic model involves a precursor conversion of ions to neutral atoms associated to ligands and then formation of dimer nuclei followed by ligand-mediated growth through ligand-associated monomer addition and autocatalytic surface growth of seed clusters. We derived kinetic equations for our model and solved them numerically using an ordinary differential equation (ODE) solver in MATLAB. We do not model diffusion and assume that NC formation is controlled by the reaction kinetics of the homogeneously mixed solution. Experimentally, rapid mixing...
of reagents can be achieved with micromixers. We use the model to predict the size distributions of NCs for a range of kinetic parameters, starting conditions, and reaction schemes.

The kinetic model explicitly shows the important role of ligands in NC nucleation and growth. The NC size distribution shifts to larger sizes with increasing ligand-associated monomer growth or autocatalytic surface growth rates. The model confirms that fast nucleation forms small clusters. A high rate of monomer formation, rapid association of ligand with metal ion/atom, and fast nucleation results in S-NCs. We show that rapidly binding ligands kinetically stabilize NCs for both strong and weak reducing agents. However, we find that fast autocatalytic growth occurs with a weak reducing agent. Consequently, large (>1.5 nm) polydisperse NCs are formed. In contrast, autocatalytic growth is prevented with a strong reducing agent and stable monodisperse S-NCs are formed.

Besides its predictive power, the model provides atomic insight into the mechanism of NC formation, which informs experiments in developing and optimizing methods to produce monodisperse NCs stabilized at early stages of growth in a homogenously mixed solution.

**METHODS**

The detailed mechanism of the kinetic model is

\[ \text{M}^+ + k_{\text{p},1} \rightarrow \text{M} \]  

(1a)

\[ \text{M} + L \xrightleftharpoons[k_{\text{d},3}]{k_{\text{a},3}} \text{ML} \]  

(1b)

\[ \text{M}^+ + L \xrightleftharpoons[k_{\text{d},2}]{k_{\text{a},2}} \text{ML}^+ \]  

(1c)

\[ \text{ML}^+ \xrightarrow{k_{\text{p},2}} \text{ML} \]  

(1d)

\[ \text{ML} + \text{ML} \xrightarrow{k_{\text{a},1}} \text{ML} \]  

(1e)

\[ \text{ML} + \text{ML} \xrightarrow{k_{\text{a},3}} \text{C}_{2,2} \]  

(1f)

\[ \text{C}_{i,j} + \text{ML} \xrightarrow[k_{\text{d},4}]{k_{\text{a},4}} \text{C}_{i+1,j+1} \]  

(1g)

\[ \text{C}_{i,j} + \text{ML} \xrightarrow[k_{\text{d},5}]{k_{\text{a},5}} \text{C}_{i+1,j+1} \]  

(1h)

\[ \text{C}_{i,j} + \text{L} \xrightleftharpoons[k_{\text{d},1}]{k_{\text{a},1}} \text{C}_{i+1,j+1} \]  

(1i)

where the cluster \( \text{C}_{i,j} = \text{M}_i \text{L}_j \) is composed of \( i \) metal atoms M and \( j \) ligands L. The reaction scheme starts with eq 1a representing the reduction of the precursor metal ion (M+) to zero-valent metal atom (M) with rate constant \( k_{\text{p},1} \). Examples of M and M+ are gold (Au) and silver (Ag) atoms and their corresponding monovalent cations. The precursor and neutral metal atom are typically solvated and coordinated with one or more ligands, for example, thiolate or phosphate. The scheme could reasonably start at eq 1d, but we explicitly consider ligand-binding in our model. Oligomers are excluded from our reaction scheme for the sake of simplicity.

In experiments, the concentration of the reducing agent is often many times the concentration of the metal precursor. For example, Luo et al. used approximately 70 equivalents of carbon monoxide as a reducing agent in the synthesis of gold thiolate NCs. Therefore, we approximate the reduction step 1a as a first-order reaction. Equations 1b and 1c show the ligand binding/unbinding with a metal atom and metal ion to form ML and ML+. With rate constants \( k_{\text{a},3} \) and \( k_{\text{d},3} \) respectively. In eq 1d, we account for the conversion of ML+ to ML with rate constant \( k_{\text{p},2} \). Similar to eq 1a, we assume a first-order reaction because of the presence of excess reducing agent.

Equations 1e and 1f illustrate the formation of the neutral dimer \( \text{C}_{2,2} \) composed of two monomers and two ligands through two pathways: irreversible dimerization of a ligand-associated monomer (ML) with itself (self-dimerization) and with a ligand-associated metal ion (ML+) (autocatalysis) with rate constants \( k_{\text{a},2} \) and \( k_{\text{a},3} \) respectively. Equation 1f is also composed of two steps: addition of a charged dimer intermediate, ML + ML+ to \( \text{C}_{2,2} \) to \( \text{C}_{2,2} \). Therefore, rate constant \( k_{\text{a},3} \) is an effective rate constant including both steps: ligand-associated ion addition and reduction of the dimer. Charged dimers and neutral dimers have been detected in the early stages of silver NC formation. Furthermore, experiments indicate that the dimer may be the nuclei for growth and is sometimes referred to as the kinetically effective nucleus.

Equations 1g and 1h show that reversible growth occurs through two pathways: addition/dissociation of ML and ML+ to/from the cluster \( \text{C}_{i,j} \) to form clusters \( \text{C}_{i+1,j+1} \) with rate constants \( k_{\text{a},4} \) and \( k_{\text{d},4} \) respectively. Equation 1h is also composed of two steps: addition of ligand-associated ion and reduction of the cluster; therefore, \( k_{\text{a},5} \) is an effective rate constant accounting for both steps. Reduction of the positively charged cluster is irreversible but there is a small probability that the ligand-associated ion dissociates from the cluster before reduction occurs. We incorporate this into the scheme above with \( k_{\text{d},5} \).

Equation 1i includes reversible ligand association/elimination step of the cluster \( \text{C}_{i,j} \) to form \( \text{C}_{i+1,j+1} \) and \( \text{C}_{i+1,j+1} \) with rate constants \( k_{\text{a},1} \) and \( k_{\text{d},1} \). These steps outline our kinetic model from which we extract three reaction schemes by modifying the rate coefficients to study NC formation.

For growth of the cluster, the model incorporates the addition of the ligand-associated metal ion (autocatalytic surface growth) and the neutral ligand-associated metal atom (classical growth) to the cluster \( \text{C}_{i,j} \) to form the cluster \( \text{C}_{i+1,j+1} \) increasing the number of monomers i and ligands j by one. These two pathways, in addition to coalescent growth, have been extensively discussed in the literature for nanoparticle formation. We did not incorporate coalescent growth into the model because coalescence of large clusters typically occurs on a longer time scale than nucleation and is prevented when ligand-binding is strong. Our interest is in the optimal conditions for the formation of sub-nanometer and nanometer size particles. Furthermore, the kinetic rate equations cannot be solved with the method of moments when ligand-mediated coalescent growth is included. Ligand binding strength and concentration are important factors in kinetically stabilizing S-NCs and NCs in their metastable stages of growth. In this regard, ligand adsorption on the cluster and elimination from it is implemented in the model through which cluster \( \text{C}_{i,j} \) gains or loses a ligand to form \( \text{C}_{i+1,j+1} \) or \( \text{C}_{i,j+1} \) respectively. Figure 1 presents a schematic of the different ways a cluster \( \text{C}_{i,j} \) can change its indices i and j and Table 1 lists a summary of the rate coefficients used in the model.
Ligand addition is determined by the number of vacant sites on a cluster of $i$ monomers, whereas ligand elimination is determined by the number of ligands $j$ already occupying those sites. If $N_{ij}$ is the total number of sites on a cluster with $i$ monomers, the number of vacant sites is $(N_{ij} - j)$ where $j$ is the number of ligands that are already in occupation. Assuming that the rate coefficients for ligand addition and elimination are linearly dependent on the respective numbers of vacant $(N_{ij} - j)$ and occupied sites $j$, we obtain the following

$$k_{g,i} = k_{g}(N_{ij} - j)$$  \hspace{1cm} (2)

$$k_{d,i} = k_{d}j$$  \hspace{1cm} (3)

$$k_{g,i,j} = k_{gac}(N_{ij} - j)$$  \hspace{1cm} (4)

$$k_{d,i,j} = k_{dac}j$$  \hspace{1cm} (5)

$$k_{b,i} = k_{b}(N_{ij} - j)$$  \hspace{1cm} (6)

$$k_{a,i} = k_{a}j$$  \hspace{1cm} (7)

The number of binding sites on an NC (eq 8) originates from an empirically derived scaling relation for the number of ligands on a cluster with $i$ monomers.\(^{28,46}\)

$$N_{ij} = [2.08i^{2/3}]$$  \hspace{1cm} (8)

The brackets in eq 8 round the number to the nearest integer. With this definition for the number of binding sites on an NC, the ratio of the number of metal atoms in the cluster to the number of ligands bound to the cluster, $i/j_{max}$ increases as a function of $i$ (Figure S1 in the Supporting Information) as expected from the coordination properties of metal atoms. Using the defined rate coefficients (eqs 1a–1i and 2–7), we derived eq 9 for the rate of change of concentration of cluster $C_{ij}$ with $i$ monomers and $j$ ligands. The minimum and maximum numbers of monomers in eq 9 are $i_{\text{min}} = 3$ and $i_{\text{max}} = 400$, respectively. The kinetic rate equations for other species $M, M^+, L, ML, ML^+$, $C_{2+}$ are listed in the Supporting Information.

$$\frac{d}{dt}[C_{ij}] = -(k_g[ML] + k_{gac}[ML^+])([C_{ij}](N_{ij} - j)) - [C_{i,j-1}](N_{ij} - j + 1) + (k_d + k_{dac})([C_{i+1,j}](j + 1) - [C_{ij}]) - k_L([C_{ij}](N_{ij} - j) - [C_{i+1,j}](N_{ij} - j + 1)) + k_L([C_{i,j+1}](j + 1) - [C_{ij}])$$  \hspace{1cm} (9)

The rate equations contain two internal coordinates $i$ and $j$, corresponding to the number of monomers $i$ and ligands $j$ in a cluster $C_{ij}$. To simplify the equations, we used the method of moments by summing over $j$ to convert each 2-D equation to two 1-D equations.\(^{28}\) Equations 2–4 define the zeroth, first, and second moments, respectively.

$$[\bar{C}_i] = \sum_{j=0}^{\infty} [C_{ij}]$$  \hspace{1cm} (10)

$$[\bar{L}_i] = \sum_{j=0}^{\infty} j[C_{ij}]$$  \hspace{1cm} (11)

$$[\bar{L}^2_i] = \sum_{j=0}^{\infty} j^2[C_{ij}]$$  \hspace{1cm} (12)

The zeroth moment (eq 10) provides the concentration of clusters with $i$ monomers irrespective of the number of ligands on the surface of the cluster. The first moment (eq 11) calculates the total concentration of ligands on clusters with $i$ monomers. The second moment (eq 12) contains information about the shape of the distribution of ligands on the clusters and can be determined if we further assume that the $j$ ligands are binomially distributed on a cluster with $i$ monomers (eqs 15 and 16), for which there is good experimental evidence.\(^{28,46}\)

Summing eq 9 over the number of ligands $j$ before and after multiplying by $j$ (eqs 10 and 11), we get two differential equations (eqs 13 and 14) for $[\bar{C}_i(t)]$ and $[\bar{L}_i(t)]$ with the time dependence suppressed for brevity.

$$\frac{d}{dt}[\bar{C}_i] = -(k_g[ML] + k_{gac}[ML^+])([\bar{C}_i]N_{ij} - [\bar{L}_i]) - ([\bar{C}_{i-1}]N_{i-1,j} - [\bar{L}_{i-1}]) + (k_d + k_{dac})([\bar{L}_{i+1}] - [\bar{L}_i])$$  \hspace{1cm} (13)

$$\frac{d}{dt}[\bar{L}_i] = -(k_g[ML] + k_{gac}[ML^+])([\bar{L}_i]N_{ij} - [\bar{C}_i]) - ([\bar{L}_{i-1}]N_{i,j-1} + [\bar{C}_{i-1}] - [\bar{C}_{i-1}]) + (k_d + k_{dac})([\bar{L}_{i+1}] - [\bar{L}_i]) - k_L([\bar{C}_i]N_{ij} - [\bar{L}_i]) - k_L[\bar{L}_i]$$  \hspace{1cm} (14)

This conversion reduces the number of equations and saves computing time in solving them numerically. To perform the sum, we assume that the number of ligands bound to a cluster with $i$ monomers follows a binomial distribution.
indicate that an excess of ligand, for example, thiolates or reducing agent, may trap NCs in their early stages of growth. Inspired by these experiments, our reaction schemes utilize a ligand-mediated kinetic model in which growth is governed by ligand-associated monomer addition to the cluster and autocatalytic surface growth (see Methods). Experiments indicate that an excess of ligand, for example, thiolates or phosphines, can shift the NC size distribution to smaller sizes and may trap NCs in their early stages of growth. Inspired by these experiments, our reaction schemes utilize a large ligand to metal ion molar ratio of 6.00 mM/0.05 mM = 120.

Table 1 presents the rate coefficients used in our model, and Table 2 displays reasonable magnitudes for three reaction schemes that are discussed in the Results and Discussion.

Table 2. Rate Coefficients Used to Create Schemes 123

<table>
<thead>
<tr>
<th>parameters</th>
<th>Scheme 1, classical</th>
<th>Scheme 2, autocatalytic</th>
<th>Scheme 3, combination</th>
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</thead>
<tbody>
<tr>
<td>$k_{d,3}/k_{a,3}$</td>
<td>$1 \times 10^3 \text{ s}^{-1}/0$</td>
<td>$1 \times 10^{-4} \text{ s}^{-1}/1 \times 10^{-3} \text{ s}^{-1}$</td>
<td>$1 \times 10^1, 1 \times 10^{-4} \text{ s}^{-1}/1 \times 10^{-3} \text{ s}^{-1}$</td>
</tr>
<tr>
<td>$k_{d,3}/k_{a,2}$</td>
<td>$1 \times 10^5 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^7 \text{ s}^{-1}$</td>
<td>$1 \times 10^5 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-7} \text{ s}^{-1}$</td>
<td>$1 \times 10^1 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-7} \text{ s}^{-1}$</td>
</tr>
<tr>
<td>$k_{d,2}/k_{a,2}$</td>
<td>$0/0$</td>
<td>$1 \times 10^5 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-7} \text{ s}^{-1}$</td>
<td>$1 \times 10^1 \text{ M}^{-1} \text{ s}^{-1}$</td>
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<tr>
<td>$k_d$</td>
<td>$1 \times 10^3, 1 \times 10^5 \text{ M}^{-1} \text{ s}^{-1}$</td>
<td>$1 \times 10^3, 1 \times 10^5 \text{ M}^{-1} \text{ s}^{-1}$</td>
<td>$1 \times 10^1, 1 \times 10^5 \text{ M}^{-1} \text{ s}^{-1}$</td>
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<tr>
<td>$k_{a,c}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$k_{d,k}$</td>
<td>$1 \times 10^5$ to $1 \times 10^6 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-9} \text{ s}^{-1}$</td>
<td>$0/0$</td>
<td>$1 \times 10^{10}$ to $1 \times 10^8 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-9} \text{ s}^{-1}$</td>
</tr>
<tr>
<td>$k_{d,k}/k_{a,k}$</td>
<td>$0/0$</td>
<td>$1 \times 10^5 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-9} \text{ s}^{-1}$</td>
<td>$1 \times 10^{10}, 1 \times 10^8 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-9} \text{ s}^{-1}$</td>
</tr>
<tr>
<td>$k_{d,k}/k_{a,k}$</td>
<td>$1 \times 10^{-3}, 1 \times 10^5 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-9} \text{ s}^{-1}$</td>
<td>$1 \times 10^5, 1 \times 10^6 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-9} \text{ s}^{-1}$</td>
<td>$1 \times 10^{-3}, 1 \times 10^6 \text{ M}^{-1} \text{ s}^{-1}/1 \times 10^{-9} \text{ s}^{-1}$</td>
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</table>

Scheme 1. Ligand-Mediated Classical Growth

$$M^+ \xrightarrow{k_{d,1}} M$$

$$M + L \xrightarrow{k_{b,1}} ML$$

$$ML + ML \xrightarrow{k_{b,2}} G_{2,2}$$

$$C_{i,j} + ML \xrightarrow{k_{b,i,j}} C_{i+1,j}$$

$$C_{i,j} + L \xrightarrow{k_{d,i,j}} C_{i,j-1}$$

Scheme 1 conforms to the classical LaMer growth mechanism. On the other hand, Scheme 2 (Table 2) has the rate constants for ligand-associated monomer nucleation $k_{d,i}$, growth $k_{d,i}$, and dissociation $k_d$ set to zero and utilizes a weak reducing agent ($k_{d,1} = 10^{-3} \text{ s}^{-1}$, $k_{d,2} = 10^{-3} \text{ s}^{-1}$), to induce slow, continuous nucleation. Scheme 2 conforms to the autocatalytic growth mechanism. Scheme 3 combines both ligand-associated monomer nucleation and growth with autocatalytic growth.

Scheme 2. Autocatalytic Surface Growth

$$M^+ \xrightarrow{k_{b,1}} M$$

$$M + L \xrightarrow{k_{b,1}} ML$$

$$M^+ + L \xrightarrow{k_{b,2}} ML^*$$

$$ML^* \xrightarrow{k_{b,3}} ML$$

$$ML + ML^* \xrightarrow{k_{b,4}} G_{2,2}$$

$$C_{i,j} + ML \xrightarrow{k_{b,i,j}} C_{i+1,j}$$

$$C_{i,j} + L \xrightarrow{k_{d,i,j}} C_{i,j-1}$$

Scheme 3. Ligand-Associated Monomer-Addition and Autocatalytic Surface Growth

$$M^+ \xrightarrow{k_{d,1}} M$$

$$M + L \xrightarrow{k_{b,1}} ML$$

$$M^+ + L \xrightarrow{k_{b,2}} ML^*$$

$$ML^* \xrightarrow{k_{b,3}} ML$$

$$ML + ML^* \xrightarrow{k_{b,4}} G_{2,2}$$

$$C_{i,j} + ML \xrightarrow{k_{b,i,j}} C_{i+1,j}$$

$$C_{i,j} + L \xrightarrow{k_{d,i,j}} C_{i,j-1}$$

□ RESULTS AND DISCUSSION

To understand the role of ligands in stabilizing NCs, we developed a ligand-mediated kinetic model in which growth is governed by ligand-associated monomer addition to the cluster and autocatalytic surface growth (see Methods). Experiments indicate that an excess of ligand, for example, thiolates or phosphines, can shift the NC size distribution to smaller sizes and may trap NCs in their early stages of growth. Inspired by these experiments, our reaction schemes utilize a large ligand to metal ion molar ratio of 6.00 mM/0.05 mM = 120.

We investigated four reaction schemes. Scheme 1 (Table 2) has the rate coefficients associated with autocatalytic growth ($k_{d,2}, k_{d,3}, k_{d,4}, k_{d,c}, k_{a,c}$) set to zero and utilizes a strong reducing agent ($k_{d,1} = 10^3 \text{ s}^{-1}$) to induce fast nucleation.
to investigate the predominance of each NC formation mechanism. Finally, Scheme 4 incorporates NC growth through the addition of a bare metal atom/ion. These calculations were done as a check on our assumption that NCs without adsorbed ligands will experience uncontrolled growth. The results show that when the ligand binding and association rates are smaller than the growth rate, the NCs grow to large sizes as the growth rate increases (Figure S4). In experiments which demonstrate NC synthesis, the metal is already bound to one or more ligands before the solution is mixed with a reducing agent. Accordingly, in Schemes 1, 2, and 3 we set the ligand binding rate to a value \(10^5\) M\(^{-1}\) s\(^{-1}\) and the NCs have larger diameters compared to the results with a strong reducing agent in Scheme 3.

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Figure 2. Scheme 1 time evolution of \([\text{ML}^+]\), \([\text{ML}], \,[\text{L}], \,[\text{C}_2], \,[\text{C}_{\text{tot}}]\), and average diameter \(D_{\text{avg}}\) for different ligand-associated monomer growth and self-dimerization rate coefficients \((k_g, k_a)\). Each graph contains plots for 18 different growth rate coefficients \(k_g\) from \(10^0\) to \(10^6\) M\(^{-1}\) s\(^{-1}\), distinguished by colors. Each row is plotted for different dimerization rate coefficients \(k_a\) \((a) 10^1, (b) 10^2,\) and \((c) 10^3\) M\(^{-1}\) s\(^{-1}\). The initial concentrations were \([\text{L}] = 6.00\) mM and \([\text{M}^+] = 0.05\) mM.

We investigated ligand-mediated single monomer (classical) growth by setting the autocatalytic growth rates \((k_{n,1}, k_{y,1})\) to zero and observing the model’s results with physically reasonable values of \(k_p, k_u, k_{gi}, k_{gy}\) and \(k_a\). Table 2 lists the rate constants used for this reaction scheme. We could not observe NC growth without a large ligand elimination rate \(k_e\). First, we discuss results for a small ligand association rate \(k_a\) \((10^{-3}\) M\(^{-1}\) s\(^{-1}\)) and the concentration of total clusters \([C_{\text{tot}}]\), and the average diameter \(D_{\text{avg}}\) (nm) of NCs for different \(k_g\) and \(k_a\) each of the reactants equilibrate within 1000 seconds for all combinations of \(k_g\) and \(k_a\) used in Figure 1. \([\text{ML}^+]\) remains zero through time because the autocatalytic rate constants are set to zero. \([\text{ML}]\) increases to a flat maximum value of 0.05 mM being produced by the \(\text{M} + \text{L}\) step and is consumed completely through self-dimerization and single monomer growth. Conversely, 6.00 mM L decreases to a flat minimum of 5.95 mM after completely associating with \(0.05\) mM M. \([\text{L}]\) then increases during NC growth before finally reaching an equilibrium concentration. The dimer concentration \([\text{C}_2]\) passes through a maximum before reaching a solubility concentration, displaying the classical LaMer curve. 21,22 The concentration of total clusters \([C_{\text{tot}}]\) and the average diameter \(D_{\text{avg}}\) increase in time up to equilibration time. Figure 2a–c shows that the average diameter \(D_{\text{avg}}\) of NCs grows larger with an increase of \(k_g\) and decreases with an increase of \(k_a\). As the rate coefficient \(k_g\) increases, more dimers are produced, providing more nuclei for growth. Consequently, the NCs have a smaller average diameter once growth is complete. The number of ligands released during growth also displays a sensitive dependence on the rate coefficient \(k_u\) and \(k_g\). As \(k_g\) increases, more ligands must be released from the NC surface for growth to occur. Therefore, the equilibrium concentration of L increases with an increase in growth rate.
Conversely, the number of free ligands decreases with an increase in $k_n$ because more dimers form.

The results of Figure 2 were calculated with a ligand association rate coefficient of $k_a = 10^{-3}$ M$^{-1}$ s$^{-1}$. We also investigated the dependence of NC sizes on $k_g$ and $k_n$ with a large ligand association rate constant ($k_a = 10^6$ M$^{-1}$ s$^{-1}$). Small and large $k_a$ correspond to slowly and rapidly binding ligands, respectively. Figure 3a again indicates increasing $k_a$ results in smaller NCs, as discussed in Figure 2. Figure 3c displays the NC size distribution for $k_a = 10^6$ M$^{-1}$ s$^{-1}$. For the same rate constants $k_g$ and $k_n$, rapidly binding ligands form smaller NCs. Figure 3b shows that increasing the growth rate $k_g$ with a slowly binding ligand shifts the size distribution to larger NCs. In contrast, Figure 3d displays the NC size distribution obtained with a rapidly binding ligand for increasing $k_g$. With a rapidly binding ligand, the NCs do not grow significantly with an increase in $k_g$ by two factors of 10. This trend reflects the limited number of binding sites on the surface of the NCs.

Scheme 2. Autocatalytic surface growth involves the addition of a charged monomer (ML$^+$) to the growing NC, while simultaneously being reduced. Therefore, we set the neutral monomer (ML) dimerization and growth and dissociation rate constants $k_g$ and $k_d$ equal to zero, which provides a model for ligand-mediated autocatalytic NC formation. Table 1 lists the rate constants used for this reaction scheme.

Similar to Figure 2, Figure 4 shows the trend of decreasing average NC diameter for increasing dimerization rate. ML$^+$ is formed through $M^+ + L$ and reaches a maximum of approximately 0.05 mM and then is consumed through autocatalytic growth and conversion to ML. Formation of ML mostly occurs through the reduction of ML$^+$, but as $k_{n,ac}$ increases from $10^1$ to $10^3$ M$^{-1}$ s$^{-1}$ the equilibrium concentration of ML decreases. Figure 4 also shows $[C_2]$ evolution with no transient maximum, which implies that autocatalytic dimerization no longer conforms to the classical LaMer paradigm in this case.

Like Scheme 1, we studied the effect of $k_{n,ac}$ and $k_{g,ac}$ on the NC size distribution. Figure 5a,b presents the trend in NC size distribution for different (a) $k_{n,ac}$ and (b) $k_{g,ac}$. Fast dimerization results in relatively smaller NCs, whereas fast growth forms relatively larger NCs in the presence of slowly smaller NCs, as discussed in Figure 2. Figure 3c displays the NC size distribution for $k_a = 10^6$ M$^{-1}$ s$^{-1}$. For the same rate constants $k_g$ and $k_n$, rapidly binding ligands form smaller NCs. Figure 3b shows that increasing the growth rate $k_g$ with a slowly binding ligand shifts the size distribution to larger NCs. In contrast, Figure 3d displays the NC size distribution obtained with a rapidly binding ligand for increasing $k_g$. With a rapidly binding ligand, the NCs do not grow significantly with an increase in $k_g$ by two factors of 10. This trend reflects the limited number of binding sites on the surface of the NCs.

Scheme 2. Autocatalytic surface growth involves the addition of a charged monomer (ML$^+$) to the growing NC, while simultaneously being reduced. Therefore, we set the neutral monomer (ML) dimerization $k_g$ and growth and dissociation rate constants $k_g$ and $k_d$ equal to zero, which provides a model for ligand-mediated autocatalytic NC formation. Table 1 lists the rate constants used for this reaction scheme.

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Like Scheme 1, we studied the effect of $k_{n,ac}$ and $k_{g,ac}$ on the NC size distribution. Figure 5a,b presents the trend in NC size distribution for different (a) $k_{n,ac}$ and (b) $k_{g,ac}$. Fast dimerization results in relatively smaller NCs, whereas fast growth forms relatively larger NCs in the presence of slowly
binding ligands. With a rapidly binding ligand (Figure 5c,d), the NCs have smaller average diameters for the same $k_n$ and $k_{ac}$ in comparison to slowly binding ligands. Similar to Scheme 1, this implies that rapidly binding ligands inhibit NC growth and trap them in early metastable stages of growth. Again, this observation is a reflection of the limited number of binding sites on the surface of an NC. In comparison to Scheme 1 and Figure 3, it is apparent that the average NC diameter is larger when a weak reducing agent is used. Furthermore, the size distribution of NCs is broadened (Figure 5) in comparison to the single monomer growth pathway with a strong reducing agent (Figure 3).

Scheme 3. In this scheme, both neutral monomer (ML) addition and autocatalytic growth pathways are active, which is a more likely scenario in real systems. In comparison to Scheme 1 and Figure 3, it is apparent that the average NC diameter is larger when a weak reducing agent is used. Furthermore, the size distribution of NCs is broadened (Figure 5) in comparison to the single monomer growth pathway with a strong reducing agent (Figure 3).

In this scheme, both neutral monomer (ML) addition and autocatalytic growth pathways are active, which is a more likely scenario in real systems. In comparison to Scheme 1 and Figure 3, it is apparent that the average NC diameter is larger when a weak reducing agent is used. Furthermore, the size distribution of NCs is broadened (Figure 5) in comparison to the single monomer growth pathway with a strong reducing agent (Figure 3).

Consequently, many nuclei (dimers) are available for growth and the average diameter of NCs is relatively small. However, in the presence of a weak reducing agent, fewer neutral monomers are produced but many charged monomers are available for autocatalytic growth. Therefore, in the presence of a weak reducing agent, the average diameter of NCs is expected to be large. In light of this, we investigated the effect of $k_{ac}$ as a switch between classical and autocatalytic growth. Classical growth predominates with large $k_{ac}$ (strong reducing agent). Conversely, autocatalytic growth predominates with small $k_{ac}$ (weak reducing agent).

Figure 6a,b shows the time-dependent concentration of different species from Scheme 3 for (a) $k_{ac} = 10^3$ s$^{-1}$ and (b) $k_{ac} = 10^4$ s$^{-1}$. Similar trends to Figure 2 are observed in Figure 6a, despite the presence of the autocatalytic growth pathway in Figure 6a. Specifically, ML shows a similar time dependence in both plots. Furthermore, $[C_{avg}]$ has regained the LaMer shape, which was not present in Figure 4 with autocatalytic growth. Similar to Figure 4, Figure 6b shows a delay in the production of ML. However, ML abruptly decreases in concentration in Figure 6b because of the presence of a strong reducing agent, and autocatalytic growth predominates with a weak reducing agent. Furthermore, the average diameter $D_{avg}$ of NCs is smaller with a strong reducing agent, which we suggest is due to the presence of more nuclei.

To elaborate on the contrasting results obtained with strong and weak reducing agents for Scheme 3, we also discuss the effects of changing the concentration of the metal ion precursor (M$^+$) and the ligand association rate to the NCs. Similar to Schemes 1 and 2, we observe smaller NCs in the presence of a rapidly binding ligand. However, when a weak reducing agent is used, the NCs are more sensitive to an increase in concentration of the metal ion precursor and the average diameter shifts to larger sizes as the concentration increases (Figure 7c,d). Figure 7a,b indicates that the concentration of the metal ion does not affect the NC size distribution using a strong reducing agent, which is likely due to the lack of NC
larger nanoparticles. Our model supports these atoms were available for autocatalytic growth, resulting in monomers were produced, but many monovalent metal nuclei for growth. At low rates, relatively few neutral reduced to neutral monomers in solution, providing many radiation dose rates. At high dose rates, most of the ions were reduced monovalent silver atoms in solution and induced silver produce solvated electrons and free radicals, which then

Figure 7e

surface of the M10L experimental results obtained through radiolytic reduction of growth.

Figure 7. Scheme 3 (a−d) NC size distribution for different concentrations of M+ and L using a fast or a slow reducing agent in the presence of a strong or a weak ligand. (e−h) Probability of finding j number of ligands on an NC given i = 10 corresponding to the rates in (a−d), respectively.

growth through the addition of a non-ligand-associated metal atom in our model.

To illustrate the effect of ligand association rate to the NC, Figure 7e−h shows the probability of finding j ligands on the surface of the M10L NC with a diameter near 0.7 nm. Figure 7c,g indicates that the probability of finding a single ligand on the surface of clusters with 10 monomers is approximately zero. On the other hand, Figure 7f,h shows that 100% of NCs with 10 monomers have more than 5 ligands on their surface. Furthermore, 40% of NCs with 10 monomers have 9 of the possible 10 binding sites covered in ligands. These observations emphasize the role of ligands in occupying binding sites on the NC surface and thereby inhibiting NC growth.

As an example, we compare the results of our calculations to experimental results obtained through radiolytic reduction of metal ions in solution. Belloni et al.51 used gamma-radiation to produce solvated electrons and free radicals, which then reduced monovalent silver atoms in solution and induced silver NC nucleation and growth. The researchers observed smaller metal clusters at high radiation dose rates compared to low dose rates. At high dose rates, most of the ions were reduced to neutral monomers in solution, providing many nuclei for growth. At low rates, relatively few neutral monomers were produced, but many monovalent metal atoms were available for autocatalytic growth, resulting in larger nanoparticles. Our model supports these findings and illustrates the NC sizes that could be obtained using a chemical reducing agent in solution (Figures 6 and 7). In this case, we emphasize that microfluidic mixers may be needed to ensure fast mixing of reactants.

As a second example, we compare the model to recent experiments on gold thiolate NCs produced through gold precursor reduction by carbon monoxide, which show a contrasting trend51 to the trend presented here. The reduction kinetics of carbon monoxide was modified by adjusting the pH of the solution. At pH 11, the reduction kinetics of carbon monoxide is faster than at pH 7. The researchers observed larger NCs (M10L10) with faster reduction kinetics at pH 11 than at pH 7 (M10L10−12).52 Our model would suggest that the opposite trend should be observed where smaller clusters are obtained with faster reduction kinetics because more nuclei are produced. To clarify the contrasting observations, we point out that gold thiolate forms oligomers in solution, and the size of the oligomer species is pH-dependent with smaller structures observed at a higher pH.52 In other words, more gold thiolate nuclei are present at higher pH and thus smaller NCs are obtained after growth completes.53 Our model does not incorporate the formation of oligomeric structures, but it is qualitatively consistent with the observation that a greater number of nuclei produce smaller NCs.

In summary, the model shows that fast nucleation, slow growth, high molar ratio of rapidly binding ligand to metal ion in a well-mixed solution promotes the formation of small NCs. Examples are the formation of gold and silver NCs and sub-NCs (S-NCs) in mixing experiments. The use of microfluidic devices could provide well-mixed solutions to facilitate NC formation.44−49 Finally, the model can be improved by incorporation of coalescent growth, diffusion, and kernels (rate constants) that account for the interaction between species in the system (e.g., DLVO theory).13,15,31 The model could be extended by incorporating metal atoms that bind to more than one ligand, for example, ML2 or ML3. Our model would predict that smaller NCs would be obtained in comparison to growth through ML as fewer surface sites would be available for growth if more than one ligand were bound to the metal monomer. This prediction is observed in experiments.

**CONCLUSIONS**

We combined ligand-mediated monomer addition and autocatalytic surface growth in a kinetic model to understand the mechanism of NC formation, which, to our knowledge, is the first study to do so. Our detailed investigation explicitly showed that fast nucleation and slow growth promotes the formation of small NCs. Our results suggest that even with a slow reducing agent, the NCs can be kinetically stabilized by ligands binding to the NC surfaces. This implies that ligands stabilize and facilitate the formation of NCs by suppressing growth and isolating NCs in their metastable states. Finally, the
kinetic model showed that, in a well-mixed solution, a high molar ratio of ligand to metal, for example, 6.00 mM/0.05 mM = 120, inhibits growth and promotes the stabilization of small NCs.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.jpcc.9b07683.

Additional equations, details about the methods, Scheme 3 results for $k_i = 10^5$, and $10^3$ M$^{-1}$ s$^{-1}$, and Scheme 4 with size distributions for various combinations of $k_i$ and $k_f$ (PDF)

■ AUTHOR INFORMATION

Corresponding Author

*E-mail: rasaihal@maine.edu.

ORCID

Mohsen Farshad: 0000-0001-5095-4361
Dylan Suvlu: 0000-0003-3216-1338
Jayendran C. Rasaiah: 0000-0002-4453-7438

Author Contributions

M.F. and D.S. contributed equally. The paper was written through contributions of all the authors. All the authors have given approval to the final version of the paper.

Notes

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■ ABBREVIATIONS

NC, nanocluster; S-NC, sub-nanometer cluster; CNT, classical nucleation theory; ODE, ordinary differential equation; FW, Finke–Watzky

■ REFERENCES


